Performance Evaluation of TOA Positioning in Asynchronous 5G Networks: A Stochastic Geometry Approach

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Abstract—The positioning of a user equipment (UE) in asynchronous 5G networks is analyzed. Three different cases of prior knowledge of the UE clock bias statistics are considered. The squared-position error bound (SPEB) for each case is derived as a function of the ideal case SPEB: no UE or base station (BS) clock biases. Analytical relationships between the SPEB of the three studied cases are established. The cumulative density function (cdf) of the SPEB for each case is analyzed numerically via Monte Carlo simulations using stochastic geometry models.

Index Terms—TOA positioning, 5G, stochastic geometry.

I. INTRODUCTION

In order to keep up with the ever more stringent wireless localization requirements set by the Federal Communications Commission (FCC) [1], wireless providers enabled network-based positioning capabilities in 4G networks and aim to improve it in upcoming 5G networks. In a network-based approach, time-of-arrival (TOA) measurements made by neighboring base stations (BSs) on the user equipment (UE) are sent to a location server that estimates the UE position. Such approaches suffer from a number of drawbacks: (i) the user’s privacy is compromised, since the UE location is revealed to the network even when there is no emergency, (ii) localization services are limited only to paying subscribers and from one cellular provider, and (iii) ambient cellular signals transmitted by other cellular providers are not exploited.

One promising technology that addresses these drawbacks and does not necessitate any change to the infrastructure is opportunistic navigation with cellular signals [2]–[4]. While more BSs can be used for positioning in an opportunistic framework, the issue of synchronization must be resolved: one must account for UE and BS biases. This challenge has been addressed for opportunistic navigation by either (i) jointly estimating these biases with the UE state, which relies on UE motion and/or sensor fusion [5]–[7], (ii) leveraging the relative frequency stability between BSs [8], or (iii) through the use of a reference receiver [9]. Moreover, several experiments have demonstrated indoor and outdoor navigation with such signals [4], [10]–[13]. The positioning and navigation performance of cellular signals has been studied for a deterministic realization of the BS positions [5], [9], [14].

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Instead of deterministic network modeling techniques, stochastic geometry has been adapted extensively to model the location of BSs over the last decade for several reasons [15], [16]: (i) modeling BS locations with point processes captures the randomness of BS deployment and (ii) stochastic geometry approach brings analytically tractable results. These perks of stochastic geometry sparked studies to characterize the localization performance in wireless networks [17], [18]. In [19], a cellular network model with a homogeneous Poisson point process (PPP) was used to derive bounds for TOA-based positioning, while the effect of signal-to-noise ratio (SNR) heterogeneity in TOA-based localization was studied in [20] by using a binomial point process (BPP) model. Such analyses shed some light on the expected localization performance in 5G networks; however, they make the impractical assumption that the UE and BSs are synchronized.

This paper aims to analyze the performance of positioning in 5G cellular systems using stochastic geometry models. The approach employed in this paper follows the one in the literature on TOA-performance characterization in wireless networks with the added complexity of accounting for UE and BS clock biases. To this end, three cases on the prior knowledge of the UE clock bias statistics are explored: (i) the UE knows its clock bias statistics, (ii) the UE does not know its clock bias statistics but estimates its clock bias along its position, and (iii) the UE does not know its clock bias statistics nor is estimating its clock bias. The third case arises when the methods proposed in the literature that assume synchronization are used in a realistic system. It is shown in this letter that this has a drastic effect on the performance. Similar to the rest of the literature, the squared-position error bound (SPEB) metric is evaluated for each case.

The remainder of the paper is organized as follows. Section II describes the BS position and TOA measurement models. Section III presents the positioning cases, characterizes their corresponding SPEBs, and establishes relationships between the SPEBs. Section IV provides numerical simulation results evaluating the cumulative density function (cdf) of the SPEBs and validating the established relationships. Concluding remarks are given in Section V.

Notation: In the rest of the paper, lower-case bold variables (e.g., $\bf{x}$) indicate column vectors and upper-case bold variables (e.g., $\bf{X}$) indicate matrices. The matrix $\bf{I}_N$ indicates the $N \times N$ identity matrix. The vector $\bf{1}_N$ indicates an $N \times 1$ vector of ones. Let $\text{diag}[x_1, \ldots, x_N]$ denote the diagonal matrix whose elements are $x_1, \ldots, x_N$. Also, let $p(z; \bf{x})$ denote the probability distribution of $z$ parameterized by $\bf{x}$, and let $\mathcal{N}(z; \bf{x}, \Sigma)$ denote the multivariate Gaussian distribution of the random vector $z$ with mean $\bf{x}$ and covariance $\Sigma$. 

Notation:
II. SYSTEM MODEL

In the following, let \( p_{\text{UE}} \triangleq [x_{\text{UE}}, y_{\text{UE}}]^T \) denote the UE’s two-dimensional (2-D) position and \( p_{\text{BS}_n} \triangleq [x_{\text{BS}_n}, y_{\text{BS}_n}]^T \) denote the \( n \)-th BS’s 2-D position, where \( n = 1, \ldots, N \), and \( N \) is the total number of available BSs. This section introduces the BS position model and the TOA measurement model for various levels of UE/BS synchronization.

A. BS Position Model

Consider a 2-D BS network modeled as a BPP, where \( N \geq 3 \) BSs are independently and uniformly distributed over an annular region centered at the origin \( o \), i.e., \( B_o(d_{\text{min}}, d_{\text{max}}) \), where \( d_{\text{min}} \) is the minimum distance required for the far-field assumption to hold and \( d_{\text{max}} \) is the maximum distance for which ranging signals can be detected by the receiver (see Fig. 1(a) for \( N = 15 \)). Such model has been proven to accurately describe the distribution of BSs in 4G and 5G networks [17]. The location of the \( n \)-th BS can be represented by \( (d_n, \phi_n) \), where \( d_n \triangleq \|p_{\text{UE}} - p_{\text{BS}_n}\|_2 \) is the range between the UE and the \( n \)-th BS and \( \phi_n \) is the bearing angle, as shown in Fig. 1(b).

![BPP realization with N = 15. (b) Parametrization of the n-th BS position by its range d_n and bearing angle φ_n to the UE.](image)

B. TOA Measurement Model

The TOA measurement made by the UE on the \( n \)-th BS, expressed in meters, can be parameterized as

\[
    z_n = d_n + c \cdot [\delta t_{\text{UE}} - \delta t_{\text{BS}_n}] + v_n, \tag{1}
\]

where \( \delta t_{\text{UE}} \) and \( \delta t_{\text{BS}_n} \) are the UE’s and the \( n \)-th BS’s clock biases, respectively, and \( v_n \) is the measurement noise, which is modeled as a zero-mean Gaussian random variable with variance \( \sigma_n^2 \). Several models of \( \sigma_n^2 \) as a function of the signal-to-noise ratio, distance, bearing angle, signal bandwidth, etc. were established [21]. The measurement noise variance also accounts for errors due to multipath propagation. In 5G applications, where the transmission bandwidths are sufficiently large, it is assumed that the first path, i.e., the direct path (DP), does not overlap with other multipath components. In this case, the DP is resolvable and the multipath signal boils down to a signal that is composed of the DP only for positioning, thereby attaining its maximum accuracy. Therefore, assuming limited interference between BSs, \( \sigma_n^2 \) is modeled as

\[
    \sigma_n^2 = \frac{c^2}{8\pi^2\beta^2} \left( \frac{d_n}{d_{\text{min}}} \right)^\alpha \frac{1}{S/N_0},
\]

where \( \beta \) is the effective signal bandwidth, \( \alpha \) is the path-loss exponent, \( S \) is the transmitted signal power, and \( N_0 \) is the power spectral density of the additive white Gaussian channel noise [20]. Moreover, it is assumed that \( \delta t_{\text{UE}} \sim \mathcal{N}(0, \sigma_{\delta t_{\text{UE}}}^2) \), \( \delta t_{\text{BS}_n} \sim \mathcal{N}(0, \sigma_{\delta t_{\text{BS}_n}}^2) \), and \( v_n \) and \( v_m \) are uncorrelated for \( n \neq m \). Equation (1) can be written in vector form as

\[
    z_N = d_N + c\delta t_{\text{UE}}1_N - c\delta t_{\text{BS}_N} + v_N, \\
    d_N \triangleq [d_1, \ldots, d_N]^T, \\
    \delta t_{\text{BS}_N} \triangleq [\delta t_{\text{BS}_1}, \ldots, \delta t_{\text{BS}_N}]^T, \\
    v_N \triangleq [v_1, \ldots, v_N]^T.
\]

Furthermore, define the following quantities

\[
    \Sigma_v \triangleq \text{cov}[\mathbf{v}_N] = \text{diag} \left[ \sigma_1^2, \ldots, \sigma_N^2 \right], \\
    \Sigma_{\delta t_{\text{BS}}} \triangleq \text{cov}[\mathbf{\delta t}_{\text{BS}_N}] = \sigma_{\delta t_{\text{BS}_N}}^2 \mathbf{I}_N, \\
    \Sigma_{\delta t_{\text{UE}}} \triangleq \sigma_{\delta t_{\text{UE}}^2} \mathbf{1}_N \mathbf{1}_N^T.
\]

III. UE POSITIONING CASES AND SPEB CHARACTERIZATION

This section analyzes three positioning cases and the SPEB is formulated for each case.

A. UE Positioning Cases

Consider the following three cases with different UE prior knowledge about its clock bias statistics:

Case I: The UE is estimating its position only and knows the statistics of its own and the BSs’ clock biases. Hence, the parameter \( \theta_I \) associated with this case and the resulting likelihood function \( p_I(z; \theta_I) \) are defined as

\[
    \theta_I \triangleq p_{\text{UE}}, \quad p_I(z; \theta_I) = \mathcal{N}(z; d, \Sigma_I), \tag{2}
\]

where \( \Sigma_I \triangleq \Sigma_v + c^2 \Sigma_{\delta t_{\text{BS}}} + c^2 \Sigma_{\delta t_{\text{UE}}} \).

Case II: The UE is estimating its position and clock bias and knows the statistics of the BSs’ clock biases only. Hence, the parameter \( \theta_{II} \) associated with this case and the resulting likelihood function \( p_{II}(z; \theta_{II}) \) are defined as

\[
    \theta_{II} \triangleq \left[ p_{\text{UE}}^T, \delta t_{\text{UE}} \right]^T, \quad p_{II}(z; \theta_{II}) = \mathcal{N}(z; d, \Sigma_{II}), \tag{3}
\]

where \( \Sigma_{II} \triangleq \Sigma_v + c^2 \Sigma_{\delta t_{\text{BS}}} \).

Case III: The UE is estimating its position only and knowing the statistics of the BSs’ clock biases only. Hence, the parameter \( \theta_{III} \) associated with this case and the resulting likelihood function \( p_{III}(z; \theta_{III}) \) are defined similarly to Case I as

\[
    \theta_{III} \triangleq \left[ \mathbf{1}, \delta t_{\text{UE}} \right]^T, \quad p_{III}(z; \theta_{III}) = p_I(z; \theta_I). \tag{4}
\]

Remark 1. The main difference between Case I and Case III is that the UE does not know the statistics of its own bias in Case III; hence, the CRLB cannot be achieved in Case III. To study Case III, an estimator that achieves the CRLB in the absence of UE clock bias is applied and its MSE is studied.

B. SPEB General Definition

The position MSE of any estimator will be lower-bounded according to \( \text{MSE} \geq \text{SPEB} \). In this paper, maximum likelihood estimators (MLEs) are used to estimate the UE position and/or clock bias from TOA measurements. Such MLEs will closely approach the CRLB, with small differences due to linearization errors. For estimators that achieve the CRLB, the position MSE becomes the SPEB; hence the choice of SPEB...
as a performance metric to be studied. It is important to note that the purpose of this paper is not to obtain an analytical expression of the distribution of the SPEB, but explicitly express the SPEB as a function of random variables whose distributions are known and have been validated (e.g., bearing angles and distances between the UE and BSSs). In addition to the fact that obtaining analytical expressions is intractable, such analysis would require a rigorous treatment that cannot fit into this letter. Instead, this work aims to characterize through Monte Carlo simulations the SPEB of the three different aforementioned cases and to draw key observations. Moreover, this paper aims to compare analytically the SPEBs of each cases in a deterministic sense, i.e., for a given BPP realization. The SPEB for each case is defined as

\[
\text{SPEB} \triangleq \text{trace} \left[ (\mathcal{I}(\theta))^{-1} \right],
\]

where \(\mathcal{I}(\theta)\) is the Fisher information matrix (FIM) of parameter \(\theta\) and \((A)_{2 \times 2}\) indicates the upper 2 \(\times\) 2 diagonal block of matrix \(A\). In the case of the FIM, this block corresponds to the two position parameters.

1) **SPEB for Case I:** From \(p_I(z; \theta_I)\), the FIM for Case I can be shown to be

\[
\mathcal{I}(\theta_I) = G^T \Sigma_I^{-1} G, \quad G \triangleq \begin{bmatrix} \cos \phi_1, \ldots, \cos \phi_N \\ \sin \phi_1, \ldots, \sin \phi_N \end{bmatrix}^T,
\]

and the SPEB is given by

\[
\text{SPEB}_I = \text{trace} \left[ (G^T \Sigma_I^{-1} G)^{-1} \right].
\]

It is worth mentioning that \(\text{SPEB}_I\) is achievable. A weighted nonlinear least-squares (WNLS) estimator with weighting matrix \(\Sigma_I^{-1}\), which is the MLE of \(\theta_I\), can achieve \(\text{SPEB}_I\).

2) **SPEB for Case II:** From \(p_H(z; \theta_H)\), the FIM for Case II can be shown to be

\[
\mathcal{I}(\theta_H) = H^T \Sigma_H^{-1} H, \quad H = [G \ 1_N].
\]

Using block matrix inversion, the SPEB for Case II can be shown to be

\[
\text{SPEB}_{HI} = \text{trace} \left[ (G^T \Psi_H G)^{-1} \right],
\]

\[
\Psi_H = \Sigma_H^{-1} - \frac{G^T \Sigma_H^{-1} G}{1_N^T \Sigma_H^{-1} 1_N}. \]

Similarly to Case I, a WNLS estimator with weighting matrix \(\Sigma_H^{-1}\) is the MLE of \(\theta_H\) and can achieve \(\text{SPEB}_{HI}\).

3) **SPEB for Case III:** For Case III, the SPEB is not calculated from the FIM, but from the estimator of Case I with \(\sigma_{d_{\text{UE}}}^2 = 0\). The term SPEB in this case becomes an abuse of notation; however, it will still be used since practically it is the best MSE the UE can achieve. For Case III, where the statistics of the UE clock bias are not known, it is assumed that the UE is employing a WNLS with weighting matrix \(\Sigma_H^{-1}\) to estimate its position. Subsequently, the SPEB for Case III can be shown to be

\[
\text{SPEB}_{HI} = \text{trace} \left[ K \Sigma_{HI} K^T \right],
\]

where \(K \triangleq (G^T \Sigma_H^{-1} G)^{-1} G^T \Sigma_H^{-1}\) and \(\Sigma_{HI} \triangleq \Sigma_I\).

### C. Performance Comparison

First, define \(\text{SPEB}_0\) as the SPEB where there is no UE clock bias, which is the case often assumed in the literature. This SPEB can be expressed as

\[
\text{SPEB}_0 = \text{trace} \left[ (G^T \Sigma_H^{-1} G)^{-1} \right].
\]

The following three lemmas establish relationships between the SPEB for each case and \(\text{SPEB}_0\).

**Lemma 1.** The SPEB for Case I can be expressed as

\[
\text{SPEB}_I = \text{SPEB}_0 + \frac{c^2 \sigma_{d_{\text{UE}}}^2}{1 + c^2 \sigma_{d_{\text{UE}}}^2} \kappa^2,
\]

for some \(\kappa \geq 0\) and \(\gamma > 0\).

**Proof.** Using the matrix inversion lemma, \(\Sigma_I^{-1}\) may be expressed as

\[
\Psi_I \triangleq \Sigma_I^{-1} = \frac{G^T \Sigma_H^{-1} G}{1_N^T \Sigma_H^{-1} G + 1_N^T \Psi_I 1_N},
\]

Using the matrix inversion lemma again, the following can be shown

\[
(G^T \Sigma_H^{-1} G)^{-1} = (G^T \Sigma_H^{-1} G)^{-1} + \frac{1}{\kappa 1_N^T \Psi_I 1_N}.
\]

The matrix \(\Psi_0\) may be expressed as

\[
\Psi_0 = \Sigma_H^{-1} G P \Sigma_H^{-1},
\]

where \(\Sigma_H^{-1}\) is a square-root of \(\Sigma_I^{-1}\) and \(P \triangleq I_N - \Sigma_H^{-1} G (G^T \Sigma_H^{-1} G)^{-1} G^T \Sigma_H^{-1}\) is an idempotent orthogonal projection matrix, i.e., \(PP = P\). Subsequently, the quadratic form \(1_N^T \Psi_0 1_N\) may be expressed as

\[
1_N^T \Psi_0 1_N = 1_N^T \Sigma_H^{-1} G P \Sigma_H^{-1} 1_N = \|P \Sigma_H^{-1} 1_N\|_2^2 \triangleq \gamma^2.
\]

It is important to note that although \(\|P \Sigma_H^{-1} 1_N\|^2 \geq 0\), it is assumed for simplicity that the trivial case is never achieved; hence \(\gamma^2 > 0\). Taking the trace of (6), using the linearity and cyclic properties of the matrix trace, and defining \(\kappa^2 \triangleq ||K 1_N||^2_2\), (5) is deduced.

**Lemma 2.** The SPEB for Case II can be expressed as

\[
\text{SPEB}_{HI} = \text{SPEB}_0 + \frac{K^T 1_N}{\gamma},
\]

for \(\gamma^2 > 0\).

**Proof.** The proof of Lemma 2 follows the same steps as in Lemma 1, for the same values of \(\gamma^2\) and \(\kappa^2\) defined earlier.

**Lemma 3.** The SPEB for Case III can be expressed as

\[
\text{SPEB}_{III} = \text{SPEB}_0 + c^2 \sigma_{d_{\text{UE}}}^2 \kappa^2,
\]

for \(\gamma^2 > 0\).

**Proof.** The proof of Lemma 3 follows the same steps as in Lemma 1, for the same value of \(\kappa^2\) defined earlier.
The lemmas stated above expose the relationships between the SPEBs of each case. It can be seen that

\[
\text{SPEB}_I \leq \text{SPEB}_H, \quad \text{SPEB}_I \leq \text{SPEB}_{III}.
\]

At the limits of \(c^2\sigma_{\delta t_{UE}}^2\), the following can be observed

\[
\lim_{\sigma_{\delta t_{UE}} \to \infty} \text{SPEB}_I = \text{SPEB}_H,
\]

\[
\lim_{\sigma_{\delta t_{UE}} \to 0} \text{SPEB}_I = \lim_{\sigma_{\delta t_{UE}} \to 0} \text{SPEB}_{III} = \text{SPEB}_0.
\]

Recall that a WNLS estimator with weighting matrix \(\Sigma_f^{-1}\) closely approaches \(\text{SPEB}_I\) for Case I, with small differences due to linearization errors. This in turn means that the aforementioned WLNS estimator, although not explicitly formulated to estimate the UE clock bias, is somehow doing so when it is given that \(\sigma_{\delta t_{UE}}^2\) is very large. On the other hand, when \(\sigma_{\delta t_{UE}}^2\) is zero, there will be a constant “loss of information” between Case I and Case II since in Case II, some information from the measurements are going to estimate a non-existing quantity.

Another interesting observation is that \(\text{SPEB}_{III}\) is not always greater than \(\text{SPEB}_H\), and the sign of the inequality between \(\text{SPEB}_{III}\) and \(\text{SPEB}_I\) depends on the values of \(\gamma^2\) and \(\sigma_{\delta t_{UE}}^2\). This result is somewhat counter-intuitive: in some cases, ignoring the existence of the UE bias yields better performance than estimating it along with the UE position. This can be explained as the tradeoff between (i) losing information on the UE position from the TOA measurements when more parameters are being estimated and (ii) additional error in the measurement. In practice however, \(c^2\sigma_{\delta t_{UE}}^2 \gg \frac{1}{d^2}\), hence \(\text{SPEB}_H\) will be practically lower than \(\text{SPEB}_{III}\).

### IV. Numerical Analysis

This section presents Monte Carlo simulation results to numerically analyze the cdf of the SPEBs in an asynchronous cellular network. For each Monte Carlo simulation, \(10^4\) realizations of the SPEB are generated to calculate the cdf.

#### A. Numerical Analysis Settings

The path-loss exponent was chosen to be \(\alpha = 3.7\) to characterize the path-loss in deep urban and indoor environments. The number of BSs was varied between 5, 10, 15, and 20, and were distributed independently and uniformly over the given area, i.e., \(B_\alpha(d_{\min}, d_{\max}) = \pi(d_{\max}^2 - d_{\min}^2)\) [15]. The Monte Carlo simulation parameters are summarized in Table I.

The 3GPP2 protocol requires BS clock biases to be bounded by \(\epsilon \sim 3\mu s\) [22]. Assuming the BS clock biases to be uniformly distributed between \(-\epsilon\) and \(\epsilon\), a moment matching method can be used to approximate a Gaussian pdf for \(\delta t_{\text{BS}}\), in order to maintain the Gaussian assumption in the analysis, leading to the values in Table I. The UE clock bias was chosen similarly. Although there are no synchronization requirements for the UE, UEs will get timing information from the servicing BS and synchronize to it. Therefore, reasonably small values of the UE clock bias are considered in the numerical analysis.

#### B. Numerical Results

Numerical results are provided next for several simulation scenarios. Note that the cdf of the PEB = \(\sqrt{\text{SPEB}}\) is provided for a more intuitive visualization.

1) **Best Case Scenario: Perfectly Synchronized Network:** The best case scenario is provided for comparison purposes. In this scenario, the UE and BS clock biases are assumed to be zero, i.e., perfectly synchronized UE-BS network. Note that in this scenario, Case I and Case III will perform similarly and Case II is expected to perform worse. The results for all cases shown in Fig. 2 for \(N = 5, 10, 15,\) and \(20\).

![Fig. 2. Cdf of PEB for Perfectly Synchronized Network](image)

2) **Effect of BS Clock Biases:** Next, the effect of BS clock biases is evaluated by setting \(N = 15\) and \(\sqrt{\text{SPEB}_{\text{UE}}} = 1\mu s\), and \(\sigma_{\delta t_{\text{BS}}}\) was varied. The results are shown in Fig. 3.

![Fig. 3. Cdf of PEB for Various BS Clock Biases](image)

The effect of BS clock bias can be seen by comparing Figs. 2 and 3: all the cdfs are shifted to the right. Moreover, Fig. 3 shows that as \(\sigma_{\delta t_{\text{BS}}}\) decreases and \(\sigma_{\text{SPEB}_{\text{UE}}}\) becomes more dominant, Cases I and II still improve significantly but Case III almost saturates since the UE bias is taking over.

3) **Effect of UE Clock Biases:** Next, the effect of UE clock bias is evaluated by setting \(N = 15\) and \(\sqrt{3\sigma_{\text{SPEB}_{\text{BS}}} = 0.25\mu s}\), and \(\sigma_{\text{SPEB}_{\text{UE}}}\) was varied. The results are shown in Fig. 4.

Fig. 4 shows how sensitive Case III is to the UE clock bias, while, as expected, the cdf of \(\text{SPEB}_H\) does not change with

<table>
<thead>
<tr>
<th>Parameter Value</th>
<th>Assumption</th>
</tr>
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<tbody>
<tr>
<td>Wireless network model</td>
<td>BPP</td>
</tr>
<tr>
<td>(N) (number of BSs)</td>
<td>5, 10, 15, 20</td>
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<tr>
<td>(\alpha) (path-loss exponent)</td>
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<tr>
<td>(S/N_0)</td>
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<tr>
<td>(d_{\max})</td>
<td>200 m</td>
</tr>
<tr>
<td>(\beta) (effective bandwidth)</td>
<td>100 MHz</td>
</tr>
</tbody>
</table>

**Table I: Parameter values for Monte Carlo simulations**

1. **Preprint of article submitted to IEEE Wireless Communications Letters**
2. **American Institute of Aeronautics and Astronautics (AIAA)**
When $\sigma_{\delta t_{UE}}$ becomes very small, Case I and Case III coincide, also as expected. The key takeaway from Fig. 4 is the importance of estimating the UE clock bias, even if some performance may be sacrificed. The difference between Case II and Case III (Case II performs better) is enormous for large values of $\sigma_{\delta t_{BS}}$, while it is only very small (Case III performs better) when $\sigma_{\delta t_{UE}}$ is small.

4) Effect of Number of BSs: Next, the effect of $N$ is evaluated. To this end, $\sqrt{\sigma_{\delta t_{BS}}}$ was fixed to 0.25 $\mu$s, $\sqrt{\sigma_{\delta t_{UE}}}$ to 1 $\mu$s, and $N$ was varied. The results are shown in Fig. 5.

Fig. 5 shows the intuitive result of the performance improving as $N$ increases. It was noticed that the PEB is decreasing as $1/\sqrt{N}$ whereas this rate is much higher (around $1/\sqrt{N}$) in the absence of biases. This reduction is due to the tradeoff that comes with adding more BSs: improvement in the UE-BS geometry at the cost of additional biases.

V. CONCLUSION

This paper evaluated the SPEB of UE positioning in asynchronous 5G networks for three cases: (i) the UE bias statistics are known and only the UE position is estimated, (ii) the UE bias statistics are unknown and UE clock bias is estimated along with its position, and (iii) the UE clock bias statistics are unknown and only the UE position is estimated. The SPEB for each case was derived as a function of the ideal case SPEB: no UE or BS clock biases. Relationships between the SPEB of the three studied cases were also established. The main takeaway from the SPEB analysis is that estimating the UE’s clock bias along with its position does not necessarily yield a better positioning performance compared to estimating the UE position only. The cdf of the SPEB for each case was evaluated numerically using stochastic geometry models. Monte Carlo simulations were presented to demonstrate the SPEB relationships between the three cases.

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