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Abstract—Autonomous ground vehicle (AGV) path planning is considered. The AGV is assumed to be equipped with receivers capable of producing pseudorange measurements to overhead global navigation satellite systems (GNSS) satellites and to cellular towers in its environment. The AGV fuses these pseudoranges to produce an estimate about its own states. The AGV is also equipped with a three-dimensional (3–D) building map of the environment. Starting from a known starting point, the AGV desires to reach a known target point by taking the shortest distance, while minimizing the AGV’s position estimation error and guaranteeing that the AGV’s position estimation uncertainty is below a desired threshold. Towards this objective, a so-called signal reliability map is first generated, which provides information about regions where large errors due to poor GNSS line-of-sight (LOS) or cellular signal multipath are expected. The vehicle uses the signal reliability map to calculate the position mean-squared error (MSE). An analytical expression for the AGV’s state estimates is derived, which is used to find an upper bound on the position bias due to multipath. An optimal path planning generation approach, which is based on Dijkstra’s algorithm, is developed to optimize the AGV’s path while minimizing the path length and position MSE, subject to keeping the position estimation uncertainty and position estimation bias due to multipath below desired thresholds. The path planning approach yields the optimal path together with a list of feasible paths and reliable GNSS satellites and cellular towers to use along these paths.

Keywords—Path planning, trajectory optimization, GNSS, cellular signals, signals of opportunity, ground vehicles, geographic information systems (GIS).

Nomenclature

- \( \alpha_p \): Node that proceeds \( \alpha \) along a path.
- \( a_i(x) \): Complex amplitude of signal path \( x \) and LTE symbol \( i \).
- \( b_{m,p} \): Multipath bias assigned to signal reliability map.
- \( c \): Speed of light.
- \( d(g) \): Cost along the path from \( s \) to \( g \).
- \( d_{LOS} \): Length of the LOS path.
- \( \text{dist}(p) \): Distance represented by location \( p \).
- \( f(\beta, \alpha) \): Path planning weight assigned to the edge between nodes \( \alpha \) and \( \beta \).
- \( g \): Target node.
- \( h_i(t) \): Channel impulse response at time \( t \) and LTE symbol \( i \).
- \( i \): LTE symbol.
- \( \lambda_{\text{max}} \): Threshold for position uncertainty.
- \( \bar{r}_{\text{max}} \): Upper bound on position bias.
- \( \beta_p \): Node that proceeds \( \beta \) along a path.
- \( \chi_m \): Multipath interference. \( \Delta \chi_{1,m}(i) + \chi_{2,m}(i) \)
- \( \delta_{\text{m-cell},m} \): \( m \)-th cellular clock bias.
- \( \delta_{\text{iono},n} \): Ionospheric delay.
- \( \delta_{\text{tropo},n} \): Tropospheric delay.
- \( \delta_{\text{cell},n} \): Experiment cellular receiver clock bias.
- \( \delta_{\text{gnss},n} \): Experiment GNSS receiver clock bias.
- \( \delta_T \): Receiver clock bias.
- \( \epsilon_m \): Constant bias between cellular clock biases.
- \( \eta_{\text{max}} \): Pseudorange bias threshold.
- \( \eta_m \): Weighted pseudorange bias threshold.
- \( \gamma(g) \): A path from nodes \( s \) to \( g \).
- \( \kappa \): Threshold for amplitude of received path.
- \( \lambda_{\text{max}}(p,t) \): Maximum eigenvalue used in path planning.
- \( G \): Sequence of all cellular signal reliability maps.
- \( M_{\text{cell}} \): Set of all paths from start to target.
- \( M_{\text{gnss}} \): Sequence of cellular signal reliability maps.
- \( M_{\text{gnss},n} \): Sequence of satellite signal reliability maps.
- \( P \): Set of all paths from start to target.
- \( T_{\text{gnss},n,p} \): Sequence of time intervals for the \( n \)-th satellite and the \( p \)-th location.
- \( \mathbf{B} \): Partition of \( \mathbf{H} \) corresponding to the clock states.
- \( \mathbf{B} = \mathbf{R}_{\alpha}^{-T} \mathbf{B} \).
- \( \Gamma = (\mathbf{\bar{B}}^T \mathbf{B})^{-1} \).
- \( \mathbf{G} = (\mathbf{I} - \mathbf{B} \mathbf{\bar{B}}^T) \mathbf{G} \).
- \( \mathbf{G} \): Partition of \( \mathbf{H} \) corresponding to the position states.
- \( \mathbf{G} = \mathbf{R}_{\alpha}^{-T} \mathbf{G} \).
- \( \mathbf{G}_{\text{cell}} \): Partition of \( \mathbf{G} \) corresponding to the cellular measurements.
- \( \mathbf{G}_{\text{gnss}} \): Partition of \( \mathbf{G} \) corresponding to the satellite measurements.

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Autonomous ground vehicles (AGVs) are predicted to improve the quality of life by automating the monotonous task of driving, while reducing crash fatalities due to human error. Considering the breadth of applications AGVs could revolutionize (e.g., cargo delivery, taxi services, emergency response, intelligent farming, etc.), a myriad number of corporations have been investing in AGV enabling technologies [1].

In light of recent tragedies [2], it is evident that AGVs need extremely reliable sensing and navigation systems. Virtually all current vehicular navigation systems rely on global navigation satellite systems (GNSS). While GNSS provides an accurate position estimate with respect to a global frame, its signals are unreliable for the safety critical application of autonomous driving. On one hand, GNSS signals are susceptible to unintentional interference, intentional jamming, and malicious spoofing [3]. On the other hand, GNSS signals are severely attenuated in deep urban canyons. Urban high-rise structures block, shadow, and reflect signals from GNSS satellites. This makes locales at which reliable and accurate GNSS position estimates are achievable to be rather spotty in urban environments [4].

To overcome the limitations of GNSS, current vehicular navigation systems fuse GNSS receivers with a suite of sensor-based technologies (e.g., inertial measurement unit (IMU), lidar, and camera) [5], [6]. High-grade sensors may violate cost, size, weight, and power (C-SWaP) constraints. Also, these dead reckoning (DR)-type sensors need an external aiding source to account for their accumulated error, only provide local position estimates, may not properly function in all environments (e.g., fog, snow, rain, dust, etc.), and are still susceptible to malicious attacks [7].

Signal-based technologies alleviate some of the shortcomings of sensor-based technologies and provide a global position estimate, but some require installing dedicated infrastructure (e.g., pseudolites [8]), while others produce a coarse position estimate (e.g., digital television signals [9]). Among signal-based technologies, cellular signals are very attractive in urban environments due to their inherent attributes [10]: (i) reception cost, size, weight, and power (C-SWaP) constraints. Also, these dead reckoning (DR)-type sensors need an external aiding source to account for their accumulated error, only provide local position estimates, may not properly function in all environments (e.g., fog, snow, rain, dust, etc.), and are still susceptible to malicious attacks [7].

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Due to the terrestrial nature of cellular towers, their received signals in urban environments suffer from line-of-sight (LOS)
blockage and severe multipath. This induces errors in cellular navigation observables (pseudoranges), which contaminate the position estimate, jeopardizing the safe operation of AGVs. Several multipath mitigation techniques for navigation with cellular signals have been proposed [12], [16], [19]–[22]. In addition to employing these techniques, an AGV could optimize its path to guarantee a desired level of accuracy, by choosing a path that yields acceptable multipath-induced biases in its cellular pseudoranges.

The objective of path planning, also known as motion planning or trajectory generation, is to optimize a path over a defined objective (e.g., path length, path duration, position uncertainty, etc.). Path planning has been extensively studied in different contexts. In robotics simultaneous localization and mapping (SLAM), path planning has been considered to steer the robot in the most informative direction [23] or to minimize the probability of becoming lost [24]. In intelligent vehicles, predictive and multirate reactive planning was considered in [25] by using the vehicle dynamics, position uncertainty, and obstacle volume to derive Lagrange-Euler equations and potential fields. In target tracking, path planning was considered to minimize the time between observations of a target made by several mobile unmanned aerial vehicles (UAVs) [26]. A cooperative urban environment target tracking algorithm that uses UAVs and AGVs and accounts for obstacles was proposed in [27], where the objective was to maximize the time that a target is observed. Path planning in spatiotemporal signal landscape maps to maximize the information gathered from ambient signals to yield accurate positioning and timing was considered in [28]–[31].

This paper and its sequel consider the following problem, originally defined in [32]. An AGV is equipped with receivers capable of producing pseudoranges to overhead GNSS satellites and to cellular long-term evolution (LTE) towers in its environment. The AGV fuses these pseudoranges to produce an estimate about its own state. The AGV is also equipped with a three-dimensional (3–D) building map of the environment. Starting from a known starting point, the AGV desires to reach a known target point by taking the shortest distance, while minimizing the AGV’s position estimation error and guaranteeing that the AGV’s position estimation uncertainty is below a desired threshold. Towards this objective, a so-called signal reliability map is first generated that specifies which signals are reliable to use at different points in the environment. Each point in the environment has a corresponding signal reliability (a boolean measure) for the different overhead GNSS satellites and nearby cellular towers. A reliable GNSS satellite means that there exists an unobstructed LOS to that satellite. A reliable cellular tower means that the pseudorange bias due to multipath is below a certain desired threshold. Next, equipped with this signal reliability map, a path planning generator produces the optimal path to the target point, if any. The path planning generator also outputs other feasible paths the AGV could take. These suboptimal, yet feasible paths could be useful, should the AGV choose to not follow the optimal path, e.g., to avoid traffic jams and road blockages due to construction or emergency. In addition, a table specifying the reliable GNSS satellites and cellular towers to use along the optimal and feasible paths is generated. Part I of the sequel focuses on modeling, analytical derivations, and algorithm development, while Part II presents comprehensive simulation and experimental results for different realistic scenarios evaluating the accuracy and efficacy of the proposed approach on a ground vehicle navigating in a deep urban environment.

The contributions of this paper are fourfold. First, the paper introduces GNSS and cellular signal reliability maps, which stores information about areas where GNSS satellites have unobstructed LOS and areas where cellular pseudorange measurements produce acceptable errors due to multipath (i.e., below a certain threshold). Second, the paper proposes a method for calculating the position mean-squared error (MSE), which measures the expected quality of the position estimate at different locations and time along the road network. Third, the paper derives an analytical expression for the position bias due to multipath, which is shown to be parameterized by a bound on the pseudorange bias and the largest eigenvalue of the position estimation error covariance. Fourth, the paper proposes a path planning method that is based on Dijkstra’s algorithm, which considers path length, position MSE, and the largest eigenvalue of the position estimation error covariance.

This paper is organized as follows. Section II formulates the path planning problem. Section III describes the GNSS and cellular measurement models and estimation framework used in this paper. Section IV presents the method for calculating GNSS and cellular signal reliability maps. Section V shows how to calculate the position MSE using the signal reliability maps, and finds an upper bound on the position bias. Section VI describes optimal path planning generation, which considers the path length, position MSE, and parameters of the upper bound on the position bias.

II. PROBLEM DESCRIPTION

This paper considers the following problem. An AGV drives in an urban environment. The AGV is equipped with receivers capable of producing pseudorange measurements on GNSS satellites and nearby cellular towers. The AGV uses these pseudorange measurements to estimate its state. The AGV desires to reach a target location by taking the shortest possible path, while guaranteeing that the uncertainty about its own position estimate is below a specified threshold (e.g., for safety concerns). A trajectory that satisfies this objective is generated either locally (i.e., within the AGV’s processor) or at a cloud-hosted path planning generator. The path planning generator uses a three-dimensional (3–D) building map of the environment to generate a so-called signal reliability map. The signal reliability map is a spatiotemporal map of the environment that measures the expected accuracy from using GNSS and cellular signals to produce an estimate of the AGV’s state. For GNSS signals, the signal reliability map specifies which GNSS satellites to which the AGV would have a clear LOS for different locations at different times in the environment. For cellular signals, the signal reliability map specifies the expected pseudorange bias due to multipath. The signal reliability maps are used to calculate the position...
MSE at each location, which in turn is used to generate an optimal path for the AGV to follow. This path is generated by minimizing the distance and MSE, while guaranteeing that the bias in the position estimate due to multipath is below a desired threshold as well as ensuring that the maximum position uncertainty is below a desired limit. In addition, the path generator produces a table of reliable GNSS satellites and cellular towers for the AGV to use as it traverses the optimal path. Fig. 1 depicts the objective of the optimal path planning generator. Here, the red circles on the street represent locations that violate the user-specified constraints (position bias or position uncertainty exceeding their respective thresholds).

III. MODEL DESCRIPTION AND ESTIMATION ALGORITHM

A. AGV-Mounted Receiver States

The AGV receives signals from M spatially-stationary cellular towers. It is assumed that the coordinates of the cellular towers are known a priori (e.g., via radio mapping or satellite images [34], [35]) and are stored locally on the AGV or on a cloud-hosted database. The 3-D position of the m-th cellular tower is denoted \( \mathbf{r}_{\text{cell},m} = \begin{bmatrix} x_{\text{cell},m}, y_{\text{cell},m}, z_{\text{cell},m} \end{bmatrix}^T \).

The AGV also receives signals from N GNSS satellites with known positions. The 3-D position of the n-th GNSS satellite is denoted \( \mathbf{r}_{\text{gnss},n} = \begin{bmatrix} x_{\text{gnss},n}, y_{\text{gnss},n}, z_{\text{gnss},n} \end{bmatrix}^T \).

The unknown states include the vehicle’s 3-D position \( \mathbf{r}_v = [x_r, y_r, z_r]^T \), the AGV-mounted receiver’s clock bias \( \delta_t \), and the clock bias of the M cellular towers \( \{\delta_t_{\text{cell},m}\}_{m=1}^M \). The cellular LTE technical specification requires transmitters in neighboring cells to be synchronized in phase up to 10 \( \mu \)s [36]. Many cellular providers synchronize nearby towers in a much tighter fashion as was demonstrated in recent experimental studies [13], [17]. This synchronization will be exploited in the proposed estimation framework to minimize the number of states that will be estimated. Specifically, only the clock bias of one of the towers will be estimated (referred to as the first tower, without loss of generality). The clock bias of the other cellular towers will be expressed as deviations from the clock bias of the first tower. The model of such deviation and the estimation algorithm will be discussed in the following subsections.

B. AGV Measurements

The AGV-mounted receiver makes pseudorange measurements to the N GNSS satellites. The n-th GNSS pseudorange measurement is modeled as

\[
\rho_{\text{gnss},n}(k) = \|\mathbf{r}_v(k) - \mathbf{r}_{\text{gnss},n}(k)\|_2 + c \cdot (\delta_t(k) - \delta_{t_{\text{gnss}},n}(k)) + c\delta_{t_{\text{iono}},n}(k) + c\delta_{t_{\text{tropo}},n}(k) + v_{\text{gnss},n}(k),
\]

where \( c \) is the speed of light; \( \delta_{t_{\text{iono}},n} \) and \( \delta_{t_{\text{tropo}},n} \) are known ionospheric and tropospheric delays, respectively; and \( \delta_{t_{\text{gnss}},n} \) is the known satellite clock bias. The terms \( r_{\text{gnss},n} \), \( \delta_{t_{\text{iono}},n} \), \( \delta_{t_{\text{tropo}},n} \), and \( \delta_{t_{\text{gnss}},n} \) are transmitted in the satellite’s navigation message. The term \( v_{\text{gnss},n} \) is the measurement noise, which is modeled as a zero-mean white Gaussian random sequence with variance \( \sigma_{\text{gnss},n}^2 \). The measurements noise across different satellites \( \{v_{\text{gnss},n}\}_{n=1}^N \) are assumed to be independent. The n-th GNSS pseudorange measurement is modified by subtracting the known \( \delta_{t_{\text{iono},n}}, \delta_{t_{\text{tropo},n}}, \) and \( \delta_{t_{\text{gnss},n}} \) to yield

\[
\tilde{z}_{\text{gnss},n} = \rho_{\text{gnss},n} - c\delta_{t_{\text{iono},n}} - c\delta_{t_{\text{tropo},n}} + c\delta_{t_{\text{gnss},n}} = \|\mathbf{r}_v - \mathbf{r}_{\text{gnss},n}\|_2 + c\delta_t + v_{\text{gnss},n}. \tag{1}
\]

The AGV-mounted receiver also makes pseudorange measurements to the M cellular towers. The m-th cellular pseudorange measurement is modeled as [37]

\[
\rho_{\text{cell},m}(k) = \|\mathbf{r}_v(k) - \mathbf{r}_{\text{cell},m}\|_2 + c \cdot (\delta_t(k) - \delta_{t_{\text{cell},m}}(k)) + v_{\text{cell},m}(k),
\]

Fig. 2 illustrates a flowchart of the optimal path planning generator framework developed in this paper.
where $v_{\text{cell},m}$ is the measurement noise, which is modelled as a zero-mean white Gaussian sequence with variance $\sigma_{v_{\text{cell},m}}^2$. The measurement noise across different cellular towers \( \{v_{\text{cell},m}\}_{m=1}^M \) are assumed to be independent.

By exploiting the synchronization between nearby cellular towers, the transmitter clock bias of the \( m \)-th cellular measurement can be expressed as

\[
c_{\text{cell},m}(k) = c_{\text{cell},1}(k) + \epsilon_m + v_{\text{cell},m}(k),
\]

for \( m = 2, \ldots, M \), where \( \epsilon_m \) is a deterministic constant bias and \( v_{\text{cell},m} \) is approximated as a zero-mean white noise sequence with variance \( \sigma_{v_{\text{cell},m}}^2 \). Therefore, for all cellular measurements other than the first cellular measurement, the \( m \)-th cellular pseudorange can be rewritten in terms of \( c_{\text{cell},1} \), namely,

\[
\rho_{\text{cell},m}(k) = \| r_r(k) - r_{\text{cell},m} \|_2 + c_{\text{cell},1}(k) - c_{\text{cell},m}(k),
\]

for \( m = 2, \ldots, M \), where \( v'_{\text{cell},m} \triangleq v_{\text{cell},m} - v_{\text{cell},1} \) is a zero-mean white noise sequence with variance \( \sigma_{\rho_{\text{cell},m}}^2 + \sigma_{v_{\text{cell},m}}^2 \).

Finally, using (2), the cellular pseudorange measurement (3) to the \( M \) cellular towers is modified according to

\[
z_{\text{cell},1}(k) = \| r_r(k) - r_{\text{cell},1} \|_2 + c_{\text{cell},1}(k)
\]

\[
z_{\text{cell},k} = \| r_r(k) - r_{\text{cell},m} \|_2 + c_{\text{cell},1}(k) - c_{\text{cell},m}(k),
\]

for \( m = 2, \ldots, M \). The next subsection describes an estimation procedure for \( \epsilon_m \) and \( \sigma_{\epsilon_m}^2 \).

C. Estimation of Cellular Measurement Clock Bias Perturbations

The perturbation parameters of the \( m \)-th cellular clock bias from the first cellular clock bias (cf. (2)), namely, the constant bias \( \epsilon_m \) and the variance \( \sigma_{\epsilon_m}^2 \) can be estimated by the AGV locally or assumed to be available from a cloud-hosted database. To estimate the constant bias \( \epsilon_m \) and variance \( \sigma_{\epsilon_m}^2 \), the measurements are differenced according to

\[
\rho_{\text{cell},1}(k) - \rho_{\text{cell},m}(k) = \| r_r(k) - r_{\text{cell},1} \|_2
\]

\[
- \| r_r(k) - r_{\text{cell},m} \|_2 + \epsilon_m + v_{\text{cell},1}(k) - v'_{\text{cell},m}(k).
\]

It is assumed that the differencing operation in (4) is performed in an open area where \( r_r \) is accurately estimated (e.g., while the AGV is initially stationary with clear LOS to GNSS satellites). Subsequently, define the measurement

\[
z_{\text{init},m}(k) \triangleq \rho_{\text{cell},1}(k) - \rho_{\text{cell},m}(k) - \| r_r(k) - r_{\text{cell},1} \|_2
\]

\[
+ \| r_r(k) - r_{\text{cell},m} \|_2 = \epsilon_m + v_{\text{cell},1}(k) - v_{\text{cell},m}(k) + v_{\epsilon,m}(k).
\]

Assuming the measurement noise to be ergodic, \( \epsilon_m \) and \( \sigma_{\epsilon,m}^2 \) can be estimated using a sample mean and a sample variance over \( K \) measurements, namely

\[
\hat{\epsilon}_m = \frac{1}{K} \sum_{k=1}^K z_{\text{init},m}(k)
\]

\[
\hat{\sigma}_{\epsilon,m}^2 = \left[ \frac{1}{K-1} \sum_{k=1}^K [z_{\text{init},m}(k) - \hat{\epsilon}_m]^2 \right] - \sigma_{\epsilon,1}^2 - \sigma_{\epsilon,M}^2.
\]

The value of \( K \) can be a fixed value chosen prior to the initialization, or can be determined during initialization by increasing \( K \) until the sample mean and variance converge. Experimentally, it was observed that the sample mean and variance converged in around 0.5 seconds with measurements at a sampling time \( T = 0.1 \) seconds (i.e., \( K \approx 50 \) samples).

D. Estimation of AGV States

The AGV’s state vector defined as \( x_r \triangleq [r_r^T, c_{\text{cell},1}, \ldots, c_{\text{cell},M}]^T \) is estimated from the measurement vector \( z_r \triangleq [z_{\text{gnss},1}, \ldots, z_{\text{cell},1}, \ldots, z_{\text{cell},M}]^T \) through a weighted non-linear least squares (WNLS) estimator. The measurement Jacobian used in the WNLS estimator is \( H \triangleq [G, B] \), where

\[
G \triangleq \begin{bmatrix} G_{\text{gnss}}^T & G_{\text{cell}}^T \end{bmatrix}^T,
\]

\[
G_{\text{gnss}} \triangleq \begin{bmatrix} r_r^T - r_{\text{gnss},1} & \ldots & r_r^T - r_{\text{gnss},N} \end{bmatrix}, \quad G_{\text{cell}} \triangleq \begin{bmatrix} r_r^T - r_{\text{cell},1}^T & \ldots & r_r^T - r_{\text{cell},M}^T \end{bmatrix},
\]

and

\[
B \triangleq \begin{bmatrix} 1_{N \times 1} & 0_{N \times 1} \\ 1_{M \times 1} & -1_{M \times 1} \end{bmatrix}, \quad (5)
\]

where \( 1 \) and \( 0 \) are matrices of ones and zeros, respectively. The weighting matrix in the WNLS is chosen as inverse of the measurement noise covariance \( R = \text{diag}([\sigma_{\epsilon,1}^2, \ldots, \sigma_{\epsilon,M}^2, \sigma_{\rho_{\text{cell},1}}^2, \ldots, \sigma_{\rho_{\text{cell},M}}^2]) \).

IV. SIGNAL RELIABILITY MAP GENERATION

A signal reliability map is a spatiotemporal map specifying for each location in the road network: (i) the GNSS satellite to which there is a clear LOS and (ii) the pseudorange multipath interference in urban environments is a dominant error source to which many mitigation techniques have been proposed [38]–[40]. Receiver-based multipath mitigation techniques typically require the LOS signal to be received [41],...
referred to by M [42], while more advanced techniques in NLOS conditions require specialized antennas and additional hardware [43–45].

The proposed approach in this paper will only consider GNSS satellites to which there is a clear LOS. To this end, the signal reliability map for GNSS signals stores information about whether the LOS path between the receiver and satellite is obstructed. Since GNSS satellite positions change with time, the GNSS signal reliability maps store the time intervals when a satellite is visible at a given location. The intervals are stored for each satellite and each location.

Formally, the GNSS signal reliability map for a given satellite is a sequence with P elements, where each element represents a location in the road network. The environment consists of N transmitters. The signal reliability map for the n-th satellite is

\[ M_{\text{gnss}} = \{T_{\text{gnss}, n, p}\}_{p=1}^{P} \quad \text{for } n = 1, \ldots, N. \]

Here, p represents a unique index corresponding to a particular location in the road network. Each \( T_{\text{gnss}, n, p} \) is a sequence of ordered pairs representing the start and end times for which the n-th satellite has unobstructed LOS at location p, i.e.,

\[ T_{\text{gnss}, n, p} = \{(t_{\text{start}, p, \tau}, t_{\text{end}, p, \tau})\}_{\tau=1}^{T_{\tau}}. \]

For one day, there are a total of \( T_{\tau} \) time intervals with start and end times \( t_{\text{start}, p, \tau} \) and \( t_{\text{end}, p, \tau} \), respectively. Since the satellite ephemeris data is known and due to the periodicity of GNSS satellites [46], the GNSS signal reliability map could be generated a priori and updated infrequently, whenever the environment undergoes certain changes (e.g., construction of a new building or demolition of an old one). The signal reliability map can be stored locally at the vehicle or at a cloud-hosted database. At a location p, the n-th satellite has unobstructed LOS at time t if

\[ t_{\text{start}, p, \tau} \leq t \leq t_{\text{end}, p, \tau}, \quad \text{for any } \tau = 1, \ldots, T_{\tau}. \]

The signal reliability maps for N satellites are collectively referred to by \( M_{\text{gnss}} = \{M_{\text{gnss}}\}_{n=1}^{N} \). Fig. 3 shows a visualization of GNSS signal reliability maps. Here, Fig 3 (a) shows a region in downtown Riverside, California, USA, in which signal blockage to a particular GNSS satellite is depicted as a 2-D red polygon. (b) The polygon layers corresponding to 12 different satellites overlayed to generate a “heat-type” map representing the number of satellites to which there is NLOS. This figure is obtained with ArcGIS® [33].

The cellular signal reliability map stores simulated pseudorange bias caused by multipath. The bias is found using the complex channel impulse response, which provides information about arrival time, phase, and power of each signal path. The complex channel impulse response can be readily calculated using proprietary simulation software (e.g., Wireless Insite [50]). This calculation requires knowledge about the cellular environment, including transmitter location, signal characteristics, antenna type, 3-D building map of the environment, and receiver location. This is carried out for all M cellular transmitters and different receiver locations within the environment. In what follows, the multipath bias calculation from the channel impulse response is discussed.

B. Cellular Signal Reliability Map Generation

In contrast to GNSS signals, cellular signals are often received at low elevation angles, which makes them more susceptible to multipath-induced errors. While multipath mitigation techniques for cellular signals has been an active area of research recently, multipath continues to be among the most dominating sources of error, thereby inducing a large pseudorange measurement bias. This is particularly the case whenever the reflected signal has a higher power than the LOS signal [14], [19], [49].

At each receiver location, the impulse response for the i-th LTE orthogonal frequency-division multiplexing (OFDM)
symbol is given by
\[ h_i(t) = \sum_{x=0}^{X-1} a_i(x) \delta(\tau - \tau_i(x)), \]  
(6)
where \( X \) is the number of impulses, \( a_i(x) \) corresponds to the complex-valued amplitude, and \( \tau_i(x) \) is the corresponding path delay. The complex channel impulse response (6) can be used to measure the multipath interference, \( \chi_m \equiv \chi_{1,m}(i) + \chi_{2,m}(i) \), for \( m = 1, \ldots, M \), where,
\[
\chi_{1,m}(i) = A \left| \sum_{l=0}^{L-1} \sum_{x=1}^{X-1} a_i(x)e^{-j2\pi(l/L)(\tau_i(x)/T_s + \bar{\varepsilon}_0 - \xi)} \right|^2,
\]
(7)
\[
- A \left| \sum_{l=0}^{L-1} \sum_{x=1}^{X-1} a_i(x)e^{-j2\pi(l/L)((\tau_i(x)/T_s + \bar{\varepsilon}_0 + \xi))} \right|^2,
\]
(8)
\[
\chi_{2,m}(i) = 2AR \left[ \left( \sum_{l=0}^{L-1} e^{-j2\pi(l/L)(\bar{\varepsilon}_0 - \xi)} \right)^2 \right. \\
- \left. \left( \sum_{l=0}^{L-1} e^{-j2\pi(l/L)(\bar{\varepsilon}_0 + \xi)} \right)^2 \right],
\]
where \( R[\cdot] \) denotes the real part, \( T_s \) is the sampling interval, \( 0 < \xi \leq 0.5 \) is the time shift in the LTE receiver’s tracking loop (\( \xi = 0.5 \) is chosen in this paper), \( L \) is the number of subcarrier symbols in the pilot (200 when the bandwidth is 20 MHz and the cell-specific reference signal (CRS) is used as the pilot), and \( A \) is the signal power due to antenna gain and implementation loss [16], [51]. The normalized symbol timing error \( \bar{\varepsilon}_0 \) is set to zero to assume perfect tracking. Using (7) and (8), the multipath interference \( \chi_m \) for all \( M \) cellular transmitters is determined.

The multipath bias is comprised of the multipath interference \( \chi_m \) and the NLOS bias (i.e., path delay between the first received path and the LOS path). That is, the multipath bias is given by
\[ b_{m,p} \equiv \chi_m + ct_i(0) - d_{LOS}, \]
(9)
where \( d_{LOS} \) is the length of the LOS path. If \( |a_i(x)| < \kappa \), for all \( x = 0, \ldots, X - 1 \), where \( \kappa \) is a threshold, the LTE signal is rendered too weak to be tracked and the signal reliability map assumes no cellular measurement at that location. For each cellular transmitter, the bias is stored for each location in the cellular signal reliability map. Formally, the cellular signal reliability map for the \( m \)-th transmitter is a sequence with \( P \) elements
\[ \mathcal{M}_{cell_m} = \{ b_{m,p} \}_{p=1}^P, \]
where \( b_{m,p} = \emptyset \) when the \( m \)-th cellular measurement is not received at the \( p \)-th location, where \( \emptyset \) denotes null. The signal reliability maps for \( M \) LTE transmitters are collectively referred to by \( \mathcal{M}_{cell} = \{ \mathcal{M}_{cell_m} \}_{m=1}^M \).

Fig. 4 shows a visualization of a cellular signal reliability map for a single cellular tower corresponding to the U.S. cellular provider AT&T in downtown Riverside, California. A raster feature is illustrated, where the black regions indicate that (i) the pseudorange bias due to multipath at the \( p \)-th location exceeds a threshold \( \eta_{\text{max}} = 0.5 \) m, i.e., \( b_{m,p} \geq 0.5 \) or (ii) there is no cellular measurement, i.e., \( b_{m,p} = \emptyset \) at the \( p \)-th position. The threshold \( \eta_{\text{max}} \) is the pseudorange bias threshold used in the path planning optimization problem explained in Subsection V-B and Section VI.

![Cellular LTE tower](image)

**Fig. 4.** A visualization of the cellular signal reliability map. The black regions indicate that (i) the pseudorange bias due to multipath at the \( p \)-th location exceeds a threshold \( \eta_{\text{max}} = 0.5 \) m, i.e., \( b_{m,p} \geq 0.5 \) or (ii) there is no cellular measurement, i.e., \( b_{m,p} = \emptyset \) at the \( p \)-th position. This figure is obtained with ArcGIS® [33].

V. POSITION MSE AND UNCERTAINTY CONSTRAINT CALCULATION

This section describes the formulation of the optimization function and constraints used to generate the optimal path for the AGV to follow. The optimization function involves the position MSE, which is discussed in the first subsection, while the constraint involves the largest eigenvalue of the position estimation error covariance, which is discussed in the second subsection.

In what follows, the biased and unbiased error states are formally defined based on the measurement model and the estimator. Since the measurement model is nonlinear with respect to the state vector \( x_r \), the model is linearized according to
\[ \Delta z_r = H \Delta x_r + v, \]
where the \( \Delta z_r \) is the measurement error vector, which is the difference between the measurement vector \( z_r \) and its estimate \( \hat{z}_r \); \( \Delta x_r \equiv x_r - \hat{x}_r \), i.e., \( \Delta x_r \) is the estimation error, which is the difference between \( x_r \) and the WNLS estimate \( \hat{x}_r \); and \( v \equiv [v_{gnss,1}, \ldots, v_{gnss,N}, v_{cell,1}, \ldots, v_{cell,M}]^T \). To analyze the
effect of multipath bias, a deterministic bias \( b \) is introduced in the measurement,
\[
\Delta z_r = \Delta z_r' + b,
\]
where \( \Delta z_r' = [\Delta z_{r_{\text{gps}1}}^T, \Delta z_{r_{\text{cell}}1}^T]^T \) is the unbiased measurement error vector.

The effect of the pseudorange bias on the position estimate can be found through the normal equation (see, for example, (7.67) in [52])
\[
\Delta x_r = (H^T R^{-1} H)^{-1} H^T R^{-1} \Delta z_r
\]
\[
= (H^T R^{-1} H)^{-1} H^T R^{-1} \Delta z_r'
\]
\[
= (H^T R^{-1} H)^{-1} H^T R^{-1} \Delta z_r' + (H^T R^{-1} H)^{-1} H^T R^{-1} b.
\]
Therefore, the bias in the pseudorange introduces an additive bias in the estimation error according to
\[
\Delta x_r = \Delta x_r' + x_{r_{\text{err}}},
\]
where \( x_{r_{\text{err}}} \triangleq [r_{r_{\text{err}}, c \delta t_{\text{err}}}^T] \) results from the multipath bias in the measurement, and \( \Delta x_r' \triangleq [\Delta x_{r_{\text{gps}1}}', \Delta x_{r_{\text{cell}1}}']^T \) is the unbiased state estimation error. The vector \( \delta t \) represents the vector of clock bias states. Therefore, the unbiased state estimation error and the state bias can be respectively expressed as
\[
\Delta x_r' = (H^T R^{-1} H)^{-1} H^T R^{-1} \Delta z_r',
\]
\[
x_{r_{\text{err}}} = (H^T R^{-1} H)^{-1} H^T R^{-1} b.
\]

The following subsection explains the steps for using the GNSS and cellular signal reliability maps whose generation was described in Section IV to (i) calculate the position MSE for the path planning cost function, and (ii) calculate the largest eigenvalue of the position estimation error covariance for the path planning constraint.

A. Position MSE

The position MSE is a scalar measure, which accounts for the precision and bias of an estimator [53], and is commonly used due to its mathematical tractability. It refers to the mean of the squared estimation error in position at a specified location and time, i.e.,

3-D position MSE equals \( E[\Delta r_1^T \Delta r_2] \)
\[
= \text{tr} \left[ E[\Delta r_1 \Delta r_2^T] \right]
\]
\[
= \text{tr} \left[ E[(\Delta x_r' + x_{r_{\text{err}}}) (\Delta x_r' + x_{r_{\text{err}}})^T] \right]
\]
\[
= \text{tr} \left[ E[\Delta x_r' \Delta x_r'^T] \right] + \| x_{r_{\text{err}}} \|^2_2,
\]
where \( E[\cdot] \) denotes the expected value and \( \text{tr}[\cdot] \) denotes the trace, and (11) follows from \( \Delta x_r' \) being zero-mean. The position bias is \( \| x_{r_{\text{err}}} \|^2_2 \), obtained from the first three elements of \( x_{r_{\text{err}}} \). The covariance of the unbiased position error is related to the weighted-position dilution of precision (WPDOP) according to [54]
\[
\text{WPDOP} \triangleq \sqrt{\text{tr} \left[ \text{cov}[\Delta x_r'] \right]}
\]
\[
= h_{11}^2 + h_{22}^2 + h_{33}^2,
\]
where \( h_{jj} \) is the \( j \)-th diagonal of \( (H^T R^{-1} H)^{-1} \).

The calculation of the position bias \( \| r_{r_{\text{err}}} \|_2 \) due to multipath uses the simulated LOS and pseudorange bias due to multipath, which were found in the signal reliability maps. The steps to calculate the position MSE are described next.

1) Step 1: Calculate the vector \( b \): For location \( p \) and time \( t \), there are \( N \leq \text{N} \) reliable GNSS measurements as determined by the signal reliability maps such that for each \( n = 1, \ldots, N \), where \( t \) satisfies a time interval defined in \( T_{\text{gps},p} \). Also, there are \( M \leq \text{M} \) reliable cellular measurements for all \( m = 1, \ldots, M \), such that \( b_{m,p} \) is not null and \( |b_{m,p}| \leq \eta_m \), where \( \eta_m \) is the \( m \)-th element of \( R_a^{-1} (N+M) \times 1 \eta_{\text{max}} \), where \( R_a \) is the Cholesky factor of \( R \), that is, \( R = R_a^T R_a \). The method for calculating the threshold \( \eta_{\text{max}} \) is shown in Subsection V-B.
The pseudorange bias vector is \( b = [0, 1, \ldots, b_{1,p}, \ldots, b_{M,p}]^T \).

2) Step 2: Calculate the Jacobian \( H \): The rows of \( H \) are calculated from the transmitter positions of the corresponding elements in \( b \), and the coordinates of location \( p \). It is assumed that the biased position and true position are close enough so that the measurement Jacobian for the true position is close to that of the biased position.

3) Step 3: Calculate the MSE: The position MSE is calculated from
\[
\text{3-D position MSE} \triangleq \text{WPDOP}^2 + \| r_{r_{\text{err}}} \|^2_2.
\]
The position MSE at a particular position \( p \) and time \( t \), denoted \( MSE(p,t) \), will be used in the path planning algorithm described in Section VI.

B. Uncertainty Constraint Calculation

This subsection describes the calculation of the path planning constraint on the largest eigenvalue of the position estimation error covariance. The purpose of this constraint is to restrict the AGV’s path to be within the maximum position uncertainty. To this end, the largest eigenvalue of the position-estimation error covariance will be used, which specifies the length of the largest axis of the uncertainty ellipsoid [55].

Methods for constraining the largest eigenvalue of a covariance matrix have been proposed in the path planning literature [28], [30], [56], [57]. The largest eigenvalue at a particular position \( p \) and at time \( t \), denoted \( \lambda_{\text{max}}(p,t) \), is found from the upper \( 3 \times 3 \) matrix block of \( (H^T R^{-1} H)^{-1} \), where \( H \) is calculated according to the method discussed in Subsection V-A.

This constraint is also related to a conservative upper bound on the position bias, which can be derived from the expression
\[
r_{r_{\text{err}}} = (\bar{G}^T \bar{G})^{-1} \bar{G}^T \bar{b},
\]
where
\[
\bar{G} = (I - \bar{B} \Gamma \bar{B}^T)^T \bar{G},
\]
\[
\bar{G} = R_{a}^{-T} G,
\]
\[
\Gamma = (\bar{B}^T \bar{B})^{-1},
\]
\[
\bar{b} = R_{a}^{-T} b, \quad \bar{B} = R_{a}^{-T} B.
\]
The derivation of (12) is given in Appendix A. The bias corresponding to the \( m \)-th cellular measurement is constrained such that \( |b_{m,p}| \leq \eta_m \), where \( \eta_m \) is the \( m \)-th element of \( R_a^{-1} (N+M) \times 1 \eta_{\text{max}} \) and \( \eta_{\text{max}} \) is the pseudorange bias threshold. The constraint can also be written as \( |\bar{b}| \leq \eta_{\text{max}} \)
where $|\cdot|$ corresponds to the absolute value of each element in the vector.

Subsequently, the upper bound on the position bias can be found according to

$$\|\mathbf{r}_{\text{err}}\|_2 \leq \maximize_{\|\mathbf{b}\|_2 \leq \sqrt{M} \eta_{\max}} \|\left(\mathbf{G}^\top \mathbf{G}\right)^{-1} \mathbf{G}^\top \mathbf{b}\|_2$$

which implies

$$\|\mathbf{r}_{\text{err}}\|_2 \leq \maximize_{\|\mathbf{b}\|_2 \leq \sqrt{M} \eta_{\max}} \|\left(\mathbf{G}^\top \mathbf{G}\right)^{-1} \mathbf{G}^\top \mathbf{b}\|_2 \leq \sqrt{M} \eta_{\max} \max \left[\lambda_{\max} \left(\mathbf{G}^\top \mathbf{G}\right)^{-1}\right] \triangleq \tilde{r}_{\max},$$

where $\sigma_{\max}'$ denotes the largest singular value, $\lambda_{\max}$ denotes the largest eigenvalue, and $\tilde{r}_{\max}$ denotes the upper bound on the position bias. Equation (13) is found by relaxing the box constraint to a 2-norm ball, where $\tilde{M}$ is the number of cellular measurements used; equation (14) follows from convexity of the objective function (maximum of a convex function over convex constraints has an optimal solution on the extreme points of the constraints); and equation (15) is found through change of variables $\mathbf{b} \triangleq \sqrt{M} \eta_{\max} \mathbf{b}$. The term $\lambda_{\max} \left(\mathbf{G}^\top \mathbf{G}\right)^{-1}$ denotes the maximum eigenvalue of the position estimation error covariance. The upper bound in (16) shows that the maximum eigenvalue also relates to the position error.

### VI. PATH PLANNING GENERATION

The path planning generation step utilizes the signal reliability map to prescribe an optimal path for the AGV to follow. This section describes the steps to determine the optimal path between a start position at a desired departure time and a target position. The optimal path is one that accounts for the shortest path length and the position MSE, subject to a maximum tolerable uncertainty (as measured by the largest eigenvalue of the position estimation error covariance).

To account for both position error and path length, the optimization cost function is chosen to be the sum of the position MSE along the path, multiplied by the distance between two adjacent points. The distance is explicitly considered in the cost function because only including position MSE could result in lengthy paths, e.g., paths that require the AGV to leave and re-enter the urban environment. The optimization function constraints account for the position bias due to cellular multipath as well as uncertainty about the AGV’s position estimate. The user-specified constraints are: (i) threshold for position bias $\tilde{r}_{\max}$ and (ii) threshold for position uncertainty $\tilde{\lambda}_{\max}$. The threshold $\tilde{\lambda}_{\max}$ is used as a constraint for all points $p$ and time $t$ along the AGV’s path, i.e., $\lambda_{\max}(p, t) \leq \tilde{\lambda}_{\max}$. The threshold $\tilde{\lambda}_{\max}$ is also used along with $\tilde{r}_{\max}$ to calculate a threshold on the pseudorange bias $\eta_{\max}$. The calculation of $\eta_{\max}$ can be achieved from (16) by substituting the user-specified $\tilde{r}_{\max}$ and using $\tilde{\lambda}_{\max}$ in place of $\lambda_{\max} \left(\mathbf{G}^\top \mathbf{G}\right)^{-1}$. Since $\tilde{M} \leq M$, $\tilde{M}$ is replaced with $M$ to calculate an upper bound that is independent of a particular location and is valid for the entire environment.

The path planning generation block solves a constrained optimization problem, discussed next, and returns the AGV’s prescribed path along with a list of reliable GNSS satellites and cellular towers to use along the path. As the AGV traverses this optimal path, it only uses signals from these reliable GNSS satellites and cellular towers. Note that to make the WNL problem observable, there needs to be at least either (i) $\tilde{N} \geq 4$ reliable GNSS satellite signals to estimate $\mathbf{x}_r' \triangleq [\mathbf{r}_r', \mathbf{c}\delta t_r]^\top$ or (ii) $\tilde{N} + \tilde{M} \geq 5$ reliable GNSS satellite and cellular signals to estimate $\mathbf{x}_r \triangleq [\mathbf{r}_r', \mathbf{c}\delta t_{cell,1}]^\top$, with $\tilde{M} \geq 1$.

Fig. 5 summarizes the flowchart of signal reliability map generation, position MSE and eigenvalue calculation, and path planning generation with the corresponding inputs and outputs defined in Sections IV and V.

![Fig. 5. Flow chart of signal reliability map generation, position MSE and uncertainty constraint calculation, and path planning generation.](image)

To account for distance in the optimization problem, each location $p$ is assigned a distance, for $p = 1, \ldots, P$. This distance, denoted $\text{dist}(p)$, signifies the length of the road network segment represented by the location $p$ and its adjacent location, and is based on the spatial discretization of the reliability maps. The steps to calculate the distance for locations $p = 1, \ldots, P$ are summarized in Fig. 6. Point $p_3$ in Fig. 6 shows the calculation of $\text{dist}(p_3)$ when the point is adjacent to an intersection, and Point $p_2$ in Fig. 6 shows the calculation of $\text{dist}(p_2)$ when the point is not adjacent to an intersection.

The path planning optimization problem is formulated next. Formally, a path from the start to the target location is denoted $\pi \in \mathcal{P}$, where $\mathcal{P}$ is the set of all paths. The path $\pi$ is composed of a sequence of position indices between the start position index $p_s$ and the target $p_t$, namely $\pi = \{p_s, p_1, p_2, \ldots, p_t\}$.

The optimization problem is expressed as

$$\minimize_{\pi \in \mathcal{P}} \sum_{p \in \pi} \text{dist}(p) \cdot \text{MSE}(p, t) \quad (17)$$

subject to

$$\lambda_{\max}(p, t) \leq \tilde{\lambda}_{\max} \quad \|\mathbf{r}_{\text{err}}\|_2 \leq \tilde{r}_{\max}.$$
Fig. 6. Steps to calculate dist(p) for 3-D points whose indices are p2 and p3. 
(i) For p4, which is adjacent to an intersection, the 3-D midpoint between p4 and p3 is calculated, then dist(p4) is the distance between the midpoint and the intersection center. (ii) For p2 which is not adjacent to an intersection, the midpoint between p3 and p2 is calculated, then the midpoint between p2 and p1 is calculated. Then, dist(p2) is the distance between the two calculated midpoints.

yield the optimization problem

\[
\begin{align*}
\text{minimize} & \quad \sum_{p \in P} \text{dist}(p) \cdot \text{MSE}(p, t) \\
\text{subject to} & \quad \lambda_{\text{max}}(p, t) \leq \bar{\lambda}_{\text{max}} \quad |\vec{b}| \leq 1 \lambda_{\text{max}}.
\end{align*}
\]

Note that the cost function in (18) accounts for both the position MSE and the path length. Other cost functions could be used to favor either part: position MSE versus path length, e.g., using an exponential for either term.

The optimization problem (18) resembles the problem of finding the shortest path in a weighted graph, where the roads are the edges of the graph and the path planning metric determines the weight of each edge. Several algorithms have been proposed to find the shortest path that use dynamic programming [58]–[60]. These algorithms cannot be used because the problem lacks optimal substructure due to the time-varying nature of the costs and constraints in (18). The following subsections show two possible approaches for prescribing a path to the AGV.

A. Path Planning Generator: Approach A

To simplify the optimization problem and use dynamic programming for the path planning algorithm, it will be assumed that the GNSS reliability map from the departure time through the time the AGV arrives at the target position is time-invariant, making \(\text{MSE}(p, t) = \text{MSE}(p)\) and \(\lambda_{\text{max}}(p, t) = \lambda_{\text{max}}(p)\). This assumption is reasonable for short AGV paths during which the geometry of the GNSS satellite constellation does not vary significantly. For example, a GPS satellite moves by approximately 3.57° in 15 minutes (see Appendix B).

Based on the aforementioned assumption, dynamic programming can be used in this approach. Among existing path planning algorithms, Dijkstra’s algorithm is most widely used and is recognized as a classic algorithm to find the shortest path between two arbitrary nodes of a weighted graph [61], [62]. Dijkstra’s algorithm is readily implementable with acceptable complexity; thus, it is adopted in this approach to solve the AGV path planning problem.

Assume that the AGV is driving in a region consisting of \(\nu\) intersections and \(\omega\) roads (e.g., side streets and highways). This region can be modeled by a graph \(\mathcal{G} = (\nu, \omega)\), which consists of \(\nu\) nodes and \(\omega\) edges. The path planning metric \(f(\beta, \alpha)\) assigns a non-negative real number corresponding to the weight of the edge from nodes \(\beta\) to \(\alpha\) in \(\mathcal{G}\). Based on the objective function in (18), the weight is given by the position MSE at all points from nodes \(\beta\) and \(\alpha\), denoted \(P(\beta, \alpha)\), multiplied by \(\text{dist}(p)\), i.e.,

\[
\begin{align*}
f(\beta, \alpha) = \sum_{p \in P(\beta, \alpha)} \text{dist}(p) \cdot \text{MSE}(p).
\end{align*}
\]

It is assumed that the deviation between \(\text{MSE}(p)\) and the MSE calculated at the true time is small. Based on the constraints in (18), if \(\lambda_{\text{max}}(p)\) exceeds \(\bar{\lambda}_{\text{max}}\) for \(p \in P(\beta, \alpha)\), then the edge is removed from the graph.

Dijkstra’s algorithm is implemented as follows. Let \(s\) indicate the start node at which the AGV starts, and let \(g\) indicate some target node. Let \(d(g)\) denote the cost along the path from \(s\) to \(g\), let \(S\) denote the set of edges that have already been evaluated by the algorithm, and let \(V\) denote the set of unvisited nodes. Within a path \(\gamma(g) \in \mathcal{G}\), denote \(\alpha_p\) as the predecessor of \(\alpha\) and \(\beta_p\) as the predecessor of \(\beta\). The path planning is initialized as follows

- \(d(s) = 0\)
- For each node \(\alpha\) adjacent to \(s\), set \(d(\alpha) = f(s, \alpha)\) and \(\alpha_p = s\)
- For each node \(\alpha\) such that \(\alpha \neq s\) and \(\alpha\) in not adjacent to \(s\), set \(d(\alpha) = \infty\)
- \(S = \{s\}\)

After the above initialization, the path planning algorithm outlined in Algorithm 1 is executed.

**Algorithm 1: Path Planning Algorithm**

**Input:** \(\mathcal{G}, s, g, S,\) and \(f(\beta, \alpha)\)

**Output:** \(d(g)\) and \(\gamma(g)\)

1. Find \(\alpha \in V\) that minimizes \(d(\alpha)\)
2. For each \(\beta\) adjacent to \(\alpha\)
   3. If \(d(\alpha) + f(\beta, \alpha) < d(\beta)\),
      4. \(d(\beta) = d(\alpha) + f(\beta, \alpha)\)
      5. \(\beta_p = \alpha\)
   6. Else,
      7. Do not change \(d(\beta)\) and \(\beta_p\)
9. End if
10. \(V \leftarrow V - \{\alpha\}\)
11. \(S \leftarrow S + \{\alpha\}\)
12. If \(S \neq V\),
   13. Goto Step 1
14. Else,
   15. Exit the Algorithm
16. End if

The node and edge data of the graph \(\mathcal{G}\) can be extracted from digital maps, such as the Open Street Map (OSM) database [63]. OSM is built by a community of mappers that contribute and maintain roads, trails, and railway stations information.
B. Path Planning Generator: Approach B

For long trajectories spanning long travel time, the optimization problem cannot be simplified as shown in the previous subsection, since the assumption that the reliability map is time-invariant would not hold. Therefore, subsection considers path planning for long trajectories, while accounting for satellite motion. One way to find the optimal path is through an exhaustive search over all possible paths, but this is computationally expensive. Instead, this approach prescribes the optimal path by: (a) finding the shortest paths (in distance), (b) calculating the cost along each of these paths and checking that the constraint is not violated, and (c) finding the path with the smallest cost that also does not violate the constraint. Although the prescribed path is not guaranteed to be optimal, searching over the shortest paths reduces computational complexity and avoids lengthy paths.

To calculate the shortest paths, a graph \( G \) is constructed similarly to Approach A, except the edge weights are the Euclidean distance along the corresponding road. Subsequently, the algorithm in [64] is used to calculate the shortest paths from the start node to target node in the graph \( G \). By assuming a constant speed for the AGV, denoted \( v_{\text{AGV}} \), the cost is calculated and stored for all paths that do not violate the constraint, and the path with the smallest cost is prescribed to the AGV.

VII. CONCLUSION

This paper considered the problem where an AGV equipped with GNSS and cellular receivers desires to reach a target location while taking the shortest path with minimum position MSE, while guaranteeing that the bias in the position estimate and the position uncertainty are below desired thresholds. A path planning generator prescribes a trajectory that satisfies this objective using a 3-D building map to create signal reliability maps for GNSS and cellular LTE signals. The signal reliability maps are used to calculate the position MSE and uncertainty-based constraint at each location, which in turn is used to generate an optimal path for the AGV to follow. In Part II of this study, extensive simulation and experimental results are presented details demonstrating (i) the improvement in the position RMSE by employing the proposed framework and (ii) the consistency between the simulated results with those obtained experimentally on a ground vehicle driving in downtown Riverside, California, USA.

APPENDIX A

RELATIONSHIP BETWEEN PSEUDORANGE AND POSITION BIAS

This appendix establishes the relationship between position bias and pseudorange bias (cf. (12). For this analysis, it is assumed that the biased position and true position are close enough so that the measurement Jacobians evaluated at each are approximately equal.

From (10), the relationship between pseudorange bias and the state bias is given by

\[ x_{r,\text{err}} \triangleq (\mathbf{H}^\top \mathbf{R}^{-1} \mathbf{H})^{-1} \mathbf{H}^\top \mathbf{R}^{-1} \mathbf{b}. \]

Consider the Cholesky factorization of \( \mathbf{R} = \mathbf{R}_a^{-1} \mathbf{R}_a \), and define \( \mathbf{H} \triangleq \mathbf{R}_a^{-\top} \mathbf{H} \) and \( \mathbf{b} \triangleq \mathbf{R}_a^{-\top} \mathbf{b} \), which results in

\[ x_{r,\text{err}} = (\mathbf{H}^\top \mathbf{H})^{-1} \mathbf{H}^\top \mathbf{b}. \] (19)

Recall that \( \mathbf{H} \triangleq [\mathbf{G}, \mathbf{B}] \) and \( x_{r,\text{err}} \triangleq [r_{r,\text{err}}, c \delta t_{\text{err}}]^\top \); therefore, (19) can be partitioned as

\[ \begin{bmatrix} r_{r,\text{err}} \\ c \delta t_{\text{err}} \end{bmatrix} \triangleq \begin{bmatrix} \mathbf{G}^\top \mathbf{G} & \mathbf{G}^\top \mathbf{B} \\ \mathbf{B}^\top \mathbf{G} & \mathbf{B}^\top \mathbf{B} \end{bmatrix}^{-1} \begin{bmatrix} \mathbf{G}^\top \\ \mathbf{B}^\top \end{bmatrix} \mathbf{b}. \]

An expression for \( r_{r,\text{err}} \) can be found through block matrix inversion

\[ r_{r,\text{err}} = \begin{bmatrix} \mathbf{A} & -\mathbf{A} \mathbf{G} \mathbf{B} \mathbf{G} \mathbf{B}^\top \mathbf{G} \end{bmatrix} \begin{bmatrix} \mathbf{G}^\top \\ \mathbf{B}^\top \end{bmatrix} \mathbf{b}, \] (20)

\[ \mathbf{A} \triangleq (\mathbf{G}^\top \mathbf{G} - \mathbf{G}^\top \mathbf{B} \mathbf{G} \mathbf{B}^\top \mathbf{G})^{-1}, \]

where \( \mathbf{G} \) is the radius of the middle earth orbit (MEO). Equations (21) and (22) can be combined to calculate the time it takes for the satellite to travel \( \phi \) degrees from the center of earth is given by

\[ \text{Time for satellite to travel} = \frac{\phi \times 1436}{360^\circ}. \] (21)

Using law of sines, \( \phi \) can be expressed in terms of \( \theta \) (the angle from zenith to the later satellite position from the earth surface) as follows

\[ \phi = \theta - \arcsin \left( \frac{\sin(180^\circ - \theta) r_{\text{Earth}}}{r_{\text{MEO}}} \right), \] (22)

where \( r_{\text{Earth}} \) is the radius of earth, and \( r_{\text{MEO}} \) is the radius of the middle earth orbit (MEO). Equations (21) and (22) can be combined to calculate the time it takes for the satellite to travel \( \theta \) degrees (see Fig. 7).
Fig. 7. Description of the variables used to approximate the change in satellite geometry.

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